

# Ranking with Multiple reference Points

## Efficient Elicitation and Learning Procedure

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November 22, 2018



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- 2 Ranking with Multiple reference Points (RMP)
- 3 Inferring the parameters of an RMP model
- 4 A SAT formulation for inferring an RMP Model
- 5 Experimental results
- 6 Conclusion and further research

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# Ranking alternatives/objects

## Problem

- ▶ Ranking alternative/object by preference
- ▶ e.g. ranking of cars

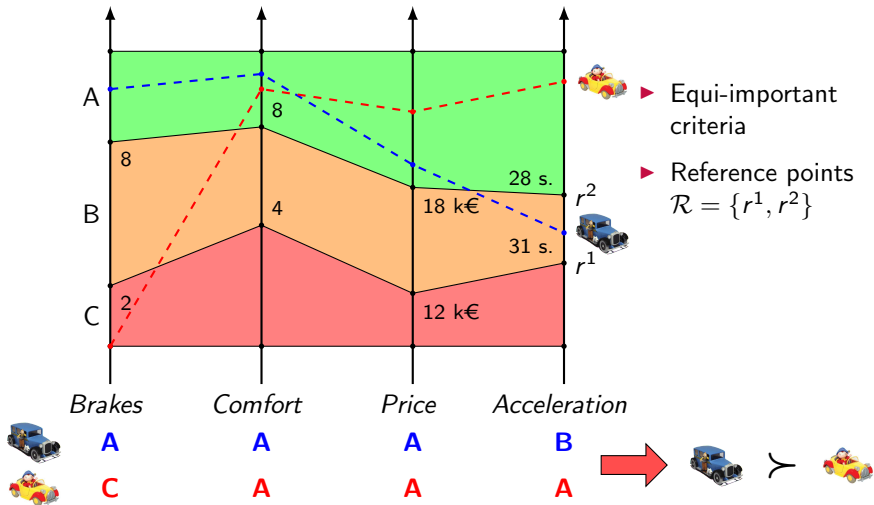


## MCDA ranking methods/models

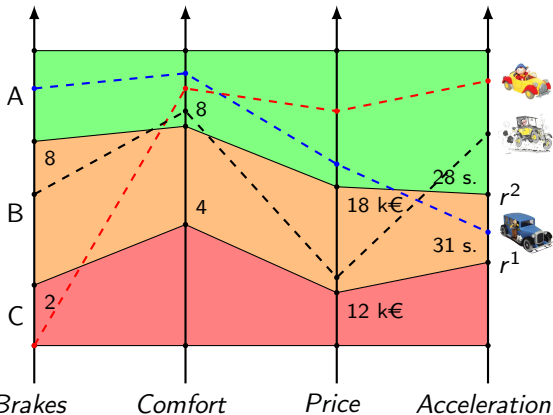
- ▶ UTilités Additives (UTA)
- ▶ ELimination and Choice Expressing REality (ELECTRE II)
- ▶ Ranking with Multiple reference Points (RMP)

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# Ranking with Multiple reference Points (RMP) I



# Ranking with Multiple reference Points (RMP) II

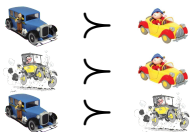


- ▶ Equi-important criteria
- ▶ Reference points  $\mathcal{R} = \{r^1, r^2\}$



Brakes      Comfort      Price      Acceleration

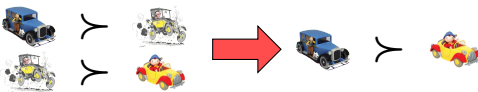
<b>A</b>	<b>A</b>	<b>A</b>	<b>B</b>
<b>C</b>	<b>A</b>	<b>A</b>	<b>A</b>
<b>B</b>	<b>A</b>	<b>B</b>	<b>A</b>



# Ranking with Multiple reference Points (RMP) III

## Some characteristics of RMP

- ▶ Model introduced by Antoine Rolland (Rolland, 2013)
- ▶ Transitivity ensured



- ▶ Safe regarding rank-reversal



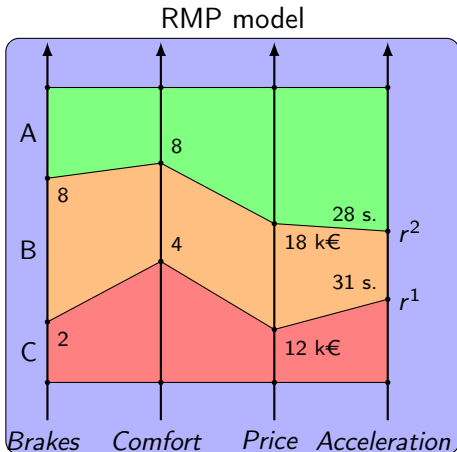
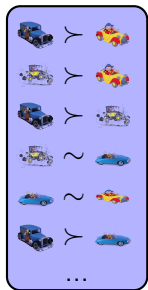
- ▶ No need for commensurate scales



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# Inferring the parameters of an RMP model I

Learning set



# Inferring the parameters of an RMP model II

## Existing algorithms

- ▶ MIP-based algorithms (Zheng et al., 2012; Liu, 2016)
  - ▶ S-RMP model (RMP with additive weights)
  - ▶ Mixed Integer Program
  - ▶ Minimization of Kemeny distance (Kemeny, 1959)
- ▶ Metaheuristic algorithm (Liu et al., 2014; Liu, 2016)
  - ▶ S-RMP model (RMP with additive weights)
  - ▶ Evolutionary algorithm
  - ▶ Reasonable computing time

## Limitations of the existing algorithms

- ▶ Additive representation of criteria importance relation
- ▶ MIP only able to handle very limited datasets
- ▶ Metaheuristic cannot always restore a S-RMP model

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# A SAT formulation for inferring an RMP Model I

## Boolean Satisfiability problem

- ▶ Boolean variables  $V$  ;
- ▶ Logical proposition about these variable  $f : \{0, 1\}^V \rightarrow \{0, 1\}$  ;
- ▶ SATisfiable if  $v^*$  exists such that  $f(v^*) = 1$
- ▶  $f$  can be expressed as conjunction of *clauses*  $\mathcal{C}$  :
 
$$f = \bigwedge_{c \in \mathcal{C}} c ;$$
- ▶ Each *clause*  $c \in \mathcal{C}$  is a disjunction of their variables or their negation :
 
$$\forall c \in \mathcal{C}, \exists c^+, c^- \in \mathcal{P}(V) : c = \bigvee_{v \in c^+} v \vee \bigvee_{v \in c^-} \neg v ;$$
- ▶ NP-complete problem BUT efficient SAT algorithms exist

## SAT for learning an RMP model

- ▶ Expression of constraints as a SAT problem
- ▶ Limited to strict preferences ( $a \succ b$ )

# A SAT formulation for inferring an RMP Model II

$$\varphi := \varphi_{\text{scales}} \wedge \varphi_{\text{profiles}} \wedge \varphi_{\text{order}} \wedge \varphi_{\text{outranking}} \wedge \varphi_{\text{lexicography}} \wedge \varphi_{\text{preference}}$$

## 1. $\varphi_{\text{scales}}$ : Monotonicity of criteria scales

$$\varphi_{\text{scales}} := \bigwedge_{i \in N} \bigwedge_{k' < k \in \mathbb{X}_i} (x_{i,h,k} \vee \neg x_{i,h,k'})$$

- ▶  $x_{i,h,k}$  : equal to 1 if value  $k$  above reference point  $r^h$  on criterion  $i$
- ▶  $N$  : set of criteria indices
- ▶  $\mathbb{X}_i$  : set of values on criterion  $i$

# A SAT formulation for inferring an RMP Model II

$$\varphi := \varphi_{\text{scales}} \wedge \varphi_{\text{profiles}} \wedge \varphi_{\text{order}} \wedge \varphi_{\text{outranking}} \wedge \varphi_{\text{lexicography}} \wedge \varphi_{\text{preference}}$$

## 2. $\varphi_{\text{profiles}}$ : Dominance of the profiles

$$\varphi_{\text{profiles}} := \varphi_{\text{profiles}_1} \wedge \varphi_{\text{profiles}_2}$$

$$\varphi_{\text{profiles}_1} := \bigwedge_{h \neq h' \in H} \bigwedge_{i \in N} \bigwedge_{k \in \mathbb{X}_i} (x_{i,h',k} \vee \neg x_{i,h,k} \vee \neg d_{h,h'})$$

$$\varphi_{\text{profiles}_2} := \bigwedge_{h < h' \in H} (d_{h,h'} \vee d_{h',h})$$

- ▶  $N$  : set of criteria indices
- ▶  $\mathbb{X}_i$  : set of values on criterion  $i$
- ▶  $H$  : set of reference points indices
- ▶  $d_{h,h'}$  : equal to 1 if value if  $r^h$  dominates  $r^{h'}$
- ▶  $x_{i,h,k}$  : equal to 1 if value  $k$  above reference point  $r^h$  on criterion  $i$

# A SAT formulation for inferring an RMP Model II

$$\varphi := \varphi_{\text{scales}} \wedge \varphi_{\text{profiles}} \wedge \varphi_{\text{order}} \wedge \varphi_{\text{outranking}} \wedge \varphi_{\text{lexicography}} \wedge \varphi_{\text{preference}}$$

### 3. $\varphi_{\text{order}}$ : Order among criteria sets

$$\varphi_{\text{order}} := \varphi_{\text{Pareto}} \wedge \varphi_{\text{completeness}} \wedge \varphi_{\text{transitivity}}$$

$$\varphi_{\text{Pareto}} := \bigwedge_{A \subseteq B \in \mathcal{P}(N)} (y_{B,A})$$

$$\varphi_{\text{completeness}} := \bigwedge_{A, B \in \mathcal{P}(N)} (y_{A,B} \vee y_{B,A})$$

$$\varphi_{\text{transitivity}} := \bigwedge_{A, B, C \in \mathcal{P}(N)} (\neg y_{A,B} \vee \neg y_{B,C} \vee y_{A,C})$$

- ▶  $\mathcal{P}(N)$  : set of possible criteria coalitions
- ▶  $y_{A,B}$  : equal to 1 if criteria coalition  $A$  is more important than criteria coalition  $B$



# A SAT formulation for inferring an RMP Model II

$$\varphi := \varphi_{\text{scales}} \wedge \varphi_{\text{profiles}} \wedge \varphi_{\text{order}} \wedge \varphi_{\text{outranking}} \wedge \varphi_{\text{lexicography}} \wedge \varphi_{\text{preference}}$$

## 4. $\varphi_{\text{outranking}}$ : Outranking relation between pairs

$$\begin{aligned} \varphi_{\text{outranking}} &:= \varphi_{\text{outranking}_1} \wedge \varphi_{\text{outranking}_2} \wedge \varphi_{\text{outranking}_3} \\ \varphi_{\text{outranking}_1} &:= \bigwedge_{A, B \in \mathcal{P}(N)} \bigwedge_{j \in J} \bigwedge_{h \in H} \bigwedge_{i \notin A} \left( \bigvee_{i \in B} \neg x_{i, h, p_i^j} \vee \bigvee_{i \in B} \neg x_{i, h, n_i^j} \vee y_{A, B} \vee \neg z_{j, h} \right) \end{aligned}$$

- ▶  $p_j \succ n_j$  : pairwise comparison  $j$
- ▶  $J$  : set of pairwise comparisons indices
- ▶  $\mathcal{P}(N)$  : set of possible criteria coalitions
- ▶  $H$  : set of reference points indices
- ▶  $x_{i, h, k}$  : equal to 1 if value  $k$  above reference point  $r^h$  on criterion  $i$
- ▶  $y_{A, B}$  : equal to 1 if criteria coalition  $A$  is more important than criteria coalition  $B$
- ▶  $z_j$  : equals to 1 if criteria set on which  $p_j$  above  $r^h$  is more important than the criteria set on which  $n_j$  is above  $r^h$

# A SAT formulation for inferring an RMP Model II

$$\varphi := \varphi_{\text{scales}} \wedge \varphi_{\text{profiles}} \wedge \varphi_{\text{order}} \wedge \varphi_{\text{outranking}} \wedge \varphi_{\text{lexicography}} \wedge \varphi_{\text{preference}}$$

## 4. $\varphi_{\text{outranking}}$ : Outranking relation between pairs

$$\begin{aligned} \varphi_{\text{outranking}} &:= \varphi_{\text{outranking}_1} \wedge \varphi_{\text{outranking}_2} \wedge \varphi_{\text{outranking}_3} \\ \varphi_{\text{outranking}_2} &:= \bigwedge_{A, B \in \mathcal{P}(N)} \bigwedge_{j \in J} \bigwedge_{h \in H} \bigwedge_{i \notin A} \left( \bigvee_{i \in B} \neg x_{i, h, p_i} \vee y_{A, B} \vee \neg z'_{j, h} \right) \end{aligned}$$

- ▶  $p_j \succ n_j$  : pairwise comparison  $j$
- ▶  $J$  : set of pairwise comparisons indices
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- ▶  $z'_j$  : equals to 1 if criteria set on which  $n_j$  above  $r^h$  is more important than the criteria set on which  $p_j$  is above  $r^h$

# A SAT formulation for inferring an RMP Model II

$$\varphi := \varphi_{\text{scales}} \wedge \varphi_{\text{profiles}} \wedge \varphi_{\text{order}} \wedge \varphi_{\text{outranking}} \wedge \varphi_{\text{lexicography}} \wedge \varphi_{\text{preference}}$$

## 4. $\varphi_{\text{outranking}}$ : Outranking relation between pairs

$$\begin{aligned} \varphi_{\text{outranking}} &:= \varphi_{\text{outranking}_1} \wedge \varphi_{\text{outranking}_2} \wedge \varphi_{\text{outranking}_3} \\ \varphi_{\text{outranking}_3} &:= \bigwedge_{A, B \in \mathcal{P}(N)} \bigwedge_{j \in J} \bigwedge_{h \in H} \bigwedge_{i \in A} (\bigvee_{i, h, p_j} \neg x_{i, h, p_j} \vee \bigvee_{i \notin A} x_{i, h, p_j} \vee \bigvee_{i \in B} \neg x_{i, h, n_j} \vee \bigvee_{i \notin B} x_{i, h, n_j} \\ &\quad \vee \neg y_{B, A} \vee z'_{j, h}) \end{aligned}$$

- ▶  $p_j \succ n_j$  : pairwise comparison  $j$
- ▶  $J$  : set of pairwise comparisons indices
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# A SAT formulation for inferring an RMP Model II

$$\varphi := \varphi_{\text{scales}} \wedge \varphi_{\text{profiles}} \wedge \varphi_{\text{order}} \wedge \varphi_{\text{outranking}} \wedge \varphi_{\text{lexicography}} \wedge \varphi_{\text{preference}}$$

## 5. $\varphi_{\text{lexicography}}$ : Lexicography of reference points

$$\varphi_{\text{lexicography}} := \varphi_{\text{lexicography}_1} \wedge \varphi_{\text{lexicography}_2} \wedge \varphi_{\text{lexicography}_3}$$

$$\varphi_{\text{lexicography}_1} := \bigwedge_{j \in J} \bigwedge_{h \leq h' \in H} (z_{j,h} \vee \neg s_{j,h'})$$

$$\varphi_{\text{lexicography}_2} := \bigwedge_{j \in J} \bigwedge_{h < h' \in H} (z'_{j,h} \vee \neg s_{j,h'})$$

$$\varphi_{\text{lexicography}_3} := \bigwedge_{h \in H} (\neg z'_{j,h} \vee \neg s_{j,h})$$

- ▶  $J$  : set of pairwise comparisons indices
- ▶  $H$  : set of reference points indices
- ▶  $z_j$  : equals to 1 if criteria set on which  $p_j$  above  $r^h$  is more important than the criteria set on which  $n_j$  is above  $r^h$
- ▶  $z'_j$  : equals to 1 if criteria set on which  $n_j$  above  $r^h$  is more important than the criteria set on which  $p_j$  is above  $r^h$
- ▶  $s_{j,h}$  : equals to 1 if  $p^j$  indifferent to  $n^j$  for all reference points  $r^{h'}$  such that  $h' < h$  and strictly outranks  $n^j$  at reference point  $r^h$

# A SAT formulation for inferring an RMP Model II

$$\varphi := \varphi_{\text{scales}} \wedge \varphi_{\text{profiles}} \wedge \varphi_{\text{order}} \wedge \varphi_{\text{outranking}} \wedge \varphi_{\text{lexicography}} \wedge \varphi_{\text{preference}}$$

## 6. $\varphi_{\text{preference}}$ : Strict preference

$$\varphi_{\text{preference}} := \bigwedge_{j \in J} \left( \bigvee_{h \in H} s_{j,h} \right)$$

- ▶  $J$  : set of pairwise comparisons indices
- ▶  $H$  : set of reference points indices
- ▶  $s_{j,h}$  : equals to 1 if  $p^j$  indifferent to  $n^j$  for all reference points  $r^{h'}$  such that  $h' < h$  and strictly outranks  $n^j$  at reference point  $r^h$

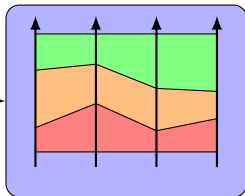
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# Experimental results I

Pairs of alternatives

(3, 5, 2, 7) ? (2, 7, 3, 9)  
 (1, 2, 3, 4) ? (4, 1, 2, 1)  
 (4, 3, 6, 5) ? (2, 7, 5, 9)  
 (5, 9, 7, 6) ? (2, 8, 3, 1)  
 (2, 6, 4, 7) ? (2, 7, 1, 4)  
 ...

$\mathcal{M}^0$



Learning set

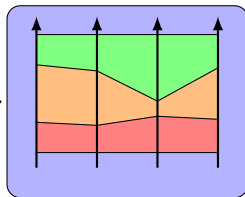
(3, 5, 2, 7)  $\succ$  (2, 7, 3, 9)  
 (1, 2, 3, 4)  $\prec$  (4, 1, 2, 1)  
 (4, 3, 6, 5)  $\succ$  (2, 7, 5, 9)  
 (5, 9, 7, 6)  $\prec$  (2, 8, 3, 1)  
 (2, 6, 4, 7)  $\succ$  (2, 7, 1, 4)  
 ...

Learning set

(3, 5, 2, 7)  $\succ$  (2, 7, 3, 9)  
 (1, 2, 3, 4)  $\succ$  (4, 1, 2, 1)  
 (4, 3, 6, 5)  $\succ$  (2, 7, 5, 9)  
 (5, 9, 7, 6)  $\prec$  (2, 8, 3, 1)  
 (2, 6, 4, 7)  $\succ$  (2, 7, 1, 4)  
 ...

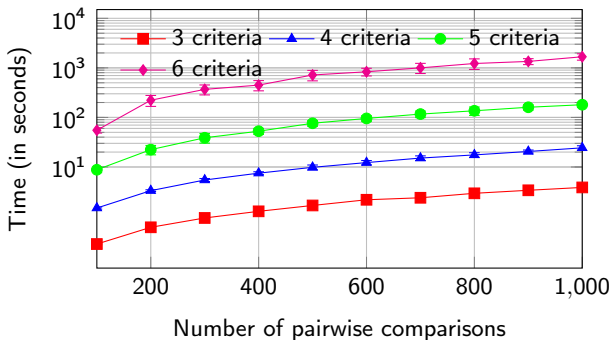
SAT-RMP

$\mathcal{M}^{\text{learned}}$



# Experimental results II

## Computing time

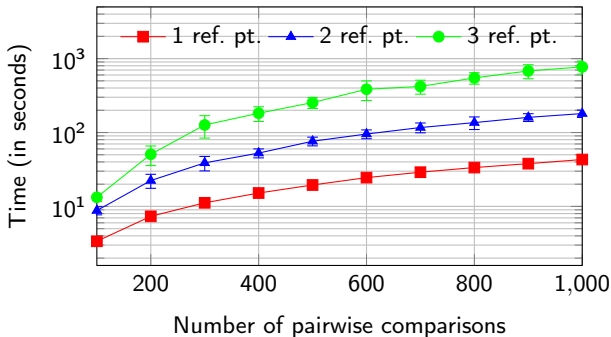


- ▶ 2 reference points
- ▶ Computing time grows fast when the number of criteria increases
- ▶ More efficient than MIP based algorithms



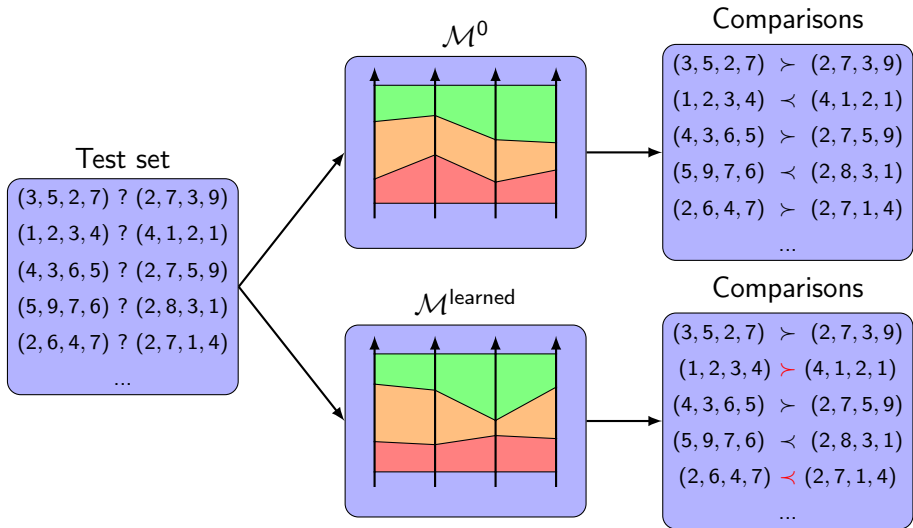
# Experimental results III

## Computing time



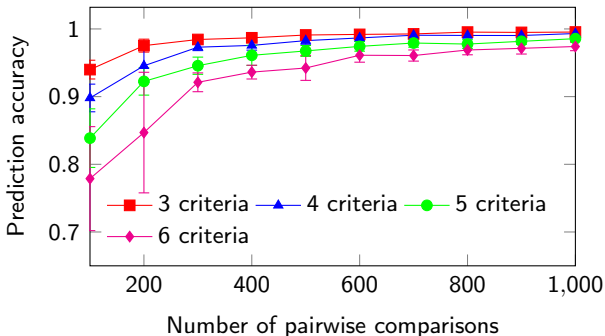
- ▶ 5 criteria
- ▶ Computing time also significantly impacted by the number of reference points

# Experimental results IV



# Experimental results V

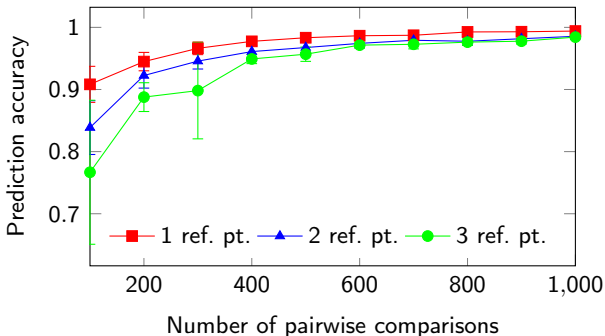
## Model retrieval



- ▶ 2 reference points
- ▶ Accuracy above 90% with barely 300 pairwise comparisons

# Experimental results VI

## Model retrieval



- ▶ 5 criteria
- ▶ Number of reference points hasn't lot of impact on the accuracy when the number of pairwise comparisons is greater than 300

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# Conclusion and further research

## Conclusion

- ▶ Efficient formulation for problems involving less than 7 criteria (1000 pairwise comparisons)

## Further research

- ▶ Support for indifference
- ▶ Formalization of SAT clauses (demonstration)
- ▶ MAXSAT formulation to handle inconsistencies



*That's all Folks!*

**Dziękuję za uwagę!**  
(Thank you for your attention!)

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