New veto rules for sorting models

Preference modeling and learning

Olivier Sobrie[1,2] - Vincent Mousseau[1] - Marc Pirlot[2]

[1]École Centrale de Paris - Laboratoire de Génie Industriel
[2]University of Mons - Faculty of engineering

July 14, 2014

Introductory example

Application

- Acceptation / Refusal of students on basis of their results

Context

- Students evaluated in 10 courses ;
- Each course has a given number of credits (ECTS) ;
- Each student is assigned in *Accepted* or *Refused*.

Conditions to be accepted

- Marks above or equal to 12/20 on at least 23 (/30) ECTS ;
- All marks at least equal to 9 with possibly one exception below 9.

Introductory example

Conditions to be accepted

- Marks above or equal to 12/20 on at least 23 (/30) ECTS ;
- All marks at least equal to 9 with possibly one exception.

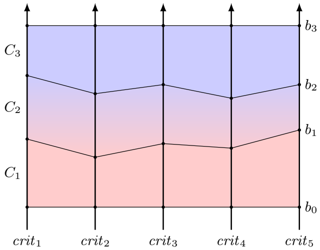
	math	physics	chemistry	biology	finance	law	management	computer sc.	sociology	marketing	accepted/refused
ECTS	4	4	4	3	3	3	3	2	2	2	
James	13	17	15	18	17	15	19	18	14	15	**A**
John	**11**	**11**	17	16	18	18	**10**	16	18	13	**R**
Michael	17	18	14	17	12	14	17	18	16	**8**	**A**
Robert	18	17	19	12	**8**	15	15	19	19	**8**	**R**

MR-Sort with veto

Principles

- Simplified version of ELECTRE TRI (no indifference and preference thresholds) ;
- Based on the concordance/discordance principle ;
- Comparison of alternatives to fixed profiles.

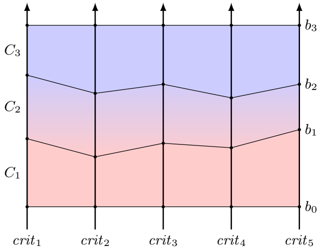
Parameters



- Profiles' performances ($b_{h,j}$ for $h = 1, ..., p - 1; j = 1, ..., n$)
- Criteria weights ($w_j \geq 0$ for $n = 1, ..., n$)
- Majority threshold (λ)
- Veto thresholds ($v_{h,j} \geq 0$ for $h = 1, ..., p - 1; j = 1, ..., n$)

MR-Sort with veto

Parameters



- Profiles' performances ($b_{h,j}$ for $h = 1, ..., p-1; j = 1, ..., n$)
- Criteria weights ($w_j \geq 0$ for $n = 1, ..., n$)
- Majority threshold (λ)
- Veto thresholds ($v_{h,j} \geq 0$ for $h = 1, ..., p-1; j = 1, ..., n$)

Assignment rule

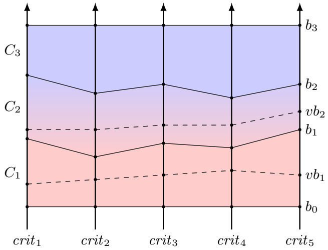
$$a \in C_h \iff a \succsim b_{h-1} \text{ and } \neg a \succsim b_h$$

$$a \succsim b_k \iff \sum_{j: a_j \geq b_{k,j}} w_j \geq \lambda \text{ and } \neg a V b_k$$

$$a V b_k \iff \exists j : a_j < b_{k,j} - v_{k,j}$$

MR-Sort with veto

Parameters



- Profiles' performances ($b_{h,j}$ for $h = 1, ..., p-1; j = 1, ..., n$)
- Criteria weights ($w_j \geq 0$ for $n = 1, ..., n$)
- Majority threshold (λ)
- Veto profiles ($vb_{h,j} \geq 0$ for $h = 1, ..., p-1; j = 1, ..., n$)

Assignment rule

$$a \in C_h \iff a \succsim b_{h-1} \text{ and } \neg a \succsim b_h$$

$$a \succsim b_k \iff \sum_{j : a_j \geq b_{k,j}} w_j \geq \lambda \text{ and } \neg a V b_k$$

$$a V b_k \iff \exists j : a_j < vb_{k,j}$$

MR-Sort with veto applied to the example

Conditions to be accepted

- Marks above or equal to 12/20 on at least 23 (/30) ECTS ;
- All marks at least equal to 9 with possibly one exception.

	math	physics	chemistry	biology	finance	law	management	computer	sociology	marketing	accepted/refused	model
ECTS	4	4	4	3	3	3	3	2	2	2		
James	13	17	15	18	17	15	19	18	14	15	A	A
John	11	11	17	16	18	18	10	16	18	13	R	R
Michael	17	18	14	17	12	14	17	18	16	8	A	R
Robert	18	17	19	12	8	15	15	19	19	8	R	R
w_j	4	4	4	3	3	3	3	2	2	2	$\sum = 30$	
$b_{1,j}$	12	12	12	12	12	12	12	12	12	12	$\lambda = 23$	
$vb_{1,j}$	9	9	9	9	9	9	9	9	9	9		

Limitations of MR-Sort with veto

Conditions to be accepted

- Mark above or equal to 12/20 on at least 23 (/30) ECTS ;
  $\Rightarrow$ Can be modeled using MR-Sort with veto
- No mark below 9/20.
  $\Rightarrow$ Can be modeled using MR-Sort with veto

Limitations of MR-Sort with veto

Conditions to be accepted

- Mark above or equal to 12/20 on at least 23 (/30) ECTS ;
  $\Rightarrow$ Can be modeled using MR-Sort with veto
- No mark below 9/20.
  $\Rightarrow$ Can be modeled using MR-Sort with veto
- **All marks at least equal to 9 with possibly one exception**
  $\Rightarrow$ **Can't be modeled with MR-Sort with veto**

Limitations of MR-Sort with veto

Conditions to be accepted

- Mark above or equal to 12/20 on at least 23 (/30) ECTS ;
  ⇒ Can be modeled using MR-Sort with veto
- No mark below 9/20.
  ⇒ Can be modeled using MR-Sort with veto
- **All marks at least equal to 9 with possibly one exception**
  ⇒ Can't be modeled with MR-Sort with veto

⇒ We propose to enrich the veto definition

Literature review - Veto

- ELECTRE TRI [Yu, 1992] method allows to take partial veto effect into account through the credibility index.
  When $a_j \leq b_{h,j} - v_{h,j}$, the assertion $a \succsim b_h$ can not hold.
- [Roy and Słowiński, 2008] proposed a new definition of ELECTRE TRI credibility index.
  It allows for "counter-veto effects" : the veto effect on some criterion is reduced when a difference in favor on an other criterion passes a counter-veto threshold.
- Other articles dealing with vetoes :
  [Perny and Roy, 1992, Perny, 1998, Fortemps and Słowiński, 2002, Bouyssou and Pirlot, 2009, Öztürk and Tsoukiàs, 2007].

Literature review - Parameters learning

- Several articles deal with learning of ELECTRE TRI parameters : [Mousseau and Słowiński, 1998, Mousseau et al., 2001, Ngo The and Mousseau, 2002, Dias et al., 2002, Dias and Mousseau, 2006].
- [Leroy et al., 2011] describe a Mixed Integer Program to learn the parameters of an MR-Sort model (without veto) on basis of assignment examples.
- [Sobrie et al., 2013] describe a metaheuristic allowing learn MR-Sort models (without veto) from large sets of assignment examples.

New veto rules

MR-Sort with veto

$$a \in C_h \iff a \succsim b_{h-1} \text{ and } \neg a \succsim b_h$$

$$a \succsim b_k \iff \sum_{j:a_j \geq b_{k,j}} w_j \geq \lambda \text{ and } \neg a V b_k$$

$$\boxed{a V b_k \iff \exists j : a_j < vb_{k,j}}$$

New veto rule : Coalitional veto

$$\boxed{a V_c b_k \iff \sum_{j:a_j < vb_{k,j}} z_j \geq \Lambda}$$

- Veto profiles : $vb_{k,j} = b_{k,j} - v_{k,j}$ for $k = 1, ..., p-1; j = 1, ..., n)$;
- Veto weights ($z_j \geq 0$ for $j = 1, ..., n$ s.t. $\sum_{j=1}^{n} z_j = 1$) ;
- Veto threshold (Λ).

Coalition veto - Consistency conditions

- For each profile b_h, the associated veto profile vb_h should be lower than b_h.
- Veto dominance : An alternative in veto with respect to the profile b_{h-1} should also be in veto w.r.t. profile b_h and all profiles above b_h

Veto dominance is guaranteed if $vb_{h,j} \geq vb_{h-1,j}, \forall h, \forall j$

New veto rules - Particular cases

General form of the new veto rule

$$aVb_k \iff \sum_{j:a_j<vb_{k,j}} z_j \geq \Lambda$$

Variant 1 : Equal veto weights

$$aVb_k \iff \sum_{j:a_j<vb_{k,j}} \frac{1}{n} \geq \Lambda$$

Variant 2 : veto weights = concordance weights

$$aVb_k \iff \sum_{j:a_j<vb_{k,j}} w_j \geq \Lambda$$

MR-Sort with coalitional veto : example

Conditions to be accepted

- Marks above or equal to 12/20 on at least 23 (/30) ECTS ;
- All marks at least equal to 9 with possibly one exception below 9.

	math	physics	chemistry	biology	finance	law	management	computer	sociology	marketing	accepted/refused	model
ECTS	4	4	4	3	3	3	3	2	2	2		
James	13	17	15	18	17	15	19	18	14	15	**A**	**A**
John	11	11	17	16	18	18	10	16	18	13	**R**	**R**
Michael	17	18	14	17	12	14	17	18	16	8	**A**	**A**
Robert	18	17	19	12	8	15	15	19	19	8	**R**	**R**
w_j	4	4	4	3	3	3	3	2	2	2	$\sum = 30$	
$b_{1,j}$	12	12	12	12	12	12	12	12	12	12	$\lambda = 23$	
$z_{1,j}$	1	1	1	1	1	1	1	1	1	1	$\sum = 10$	
$vb_{1,j}$	9	9	9	9	9	9	9	9	9	9	$\Lambda = 2$	

Learning MR-Sort model with coalitional vetoes

Input

- Example of assignments and their performances

Objective functions

1. Maximize number of alternatives compatible with the model
2. Minimize the number of vetoes

Number of parameters to learn

(n : number of criteria ; p : number of categories)

- MR-Sort without veto : $np + 1$
- MR-Sort with standard veto : $2np - n + 1$
- MR-Sort with new veto rule : $2np + 2$

Method

- All model parameters are learned at the same time
- Mixed Integer Programming

Linear programming - Constraints modeling

Condition to assign an alternative a in category C_h

$$a \in C_h \iff \begin{cases} \sum_{j:a_j \geq b_{h-1,j}} w_j \geq \lambda & \text{and } \sum_{j:a_j \leq vb_{h-1,j}} z_j < \Lambda \\ \sum_{j:a_j \geq b_{h,j}} w_j < \lambda & \text{or } \sum_{j:a_j \leq vb_{h,j}} z_j \geq \Lambda \end{cases}$$

Linear programming - Constraints modeling

Condition to assign an alternative a in category C_h

$$a \in C_h \iff \begin{cases} \sum_{j=1}^{n} c_{a,j}^{h-1} \geq \lambda & \text{and } \sum_{j=1}^{n} \mu_{a,j}^{h-1} < \Lambda \\ \sum_{j=1}^{n} c_{a,j}^{h} < \lambda & \text{or } \sum_{j=1}^{n} \mu_{a,j}^{h} \geq \Lambda \end{cases}$$

with $c_{a,j}^{l}$ and $\mu_{a,j}^{l}$ for $l = h-1, h$ such that :

$$c_{a,j}^{l} = \begin{cases} w_j & \text{if } a_j \geq b_{l,j} \\ 0 & \text{if } a_j < b_{l,j} \end{cases} \qquad \mu_{a,j}^{l} = \begin{cases} z_j & \text{if } a_j \leq b_{l,j} - v_{l,j} \\ 0 & \text{if } a_j > b_{l,j} - v_{l,j} \end{cases}$$

Linear programming - Constraints modeling

Condition to assign an alternative a in category C_h

$$a \in C_h \iff \begin{cases} \sum_{j=1}^{n} c_{a,j}^{h-1} \geq \lambda & \text{and } \sum_{j=1}^{n} \mu_{a,j}^{h-1} < \Lambda \\ \sum_{j=1}^{n} c_{a,j}^{h} < \lambda & \text{or } \sum_{j=1}^{n} \mu_{a,j}^{h} \geq \Lambda \end{cases}$$

with $c_{a,j}^{l}$ and $\mu_{a,j}^{l}$ for $l = h-1, h$ such that :

$$c_{a,j}^{l} = \begin{cases} w_j & \text{if } a_j \geq b_{l,j} \\ 0 & \text{if } a_j < b_{l,j} \end{cases} \qquad \mu_{a,j}^{l} = \begin{cases} z_j & \text{if } a_j \leq b_{l,j} - v_{l,j} \\ 0 & \text{if } a_j > b_{l,j} - v_{l,j} \end{cases}$$

To linearize these constraints, we introduce binary variables :

$$\delta_{a,j}^{l} = \begin{cases} 1 & \text{if } a_j \geq b_{l,j} \\ 0 & \text{if } a_j < b_{l,j} \end{cases} \qquad \nu_{a,j}^{l} = \begin{cases} 1 & \text{if } a_j \leq b_{l,j} - v_{l,j} \\ 0 & \text{if } a_j > b_{l,j} - v_{l,j} \end{cases}$$

$$\begin{cases} a_j - b_{l,j} & < M\delta_{a,j}^{l} \\ a_j - b_{l,j} & \geq M(\delta_{a,j}^{l} - 1) \end{cases} \qquad \begin{cases} a_j - b_{l,j} + v_{l,j} & > -M\nu_{a,j}^{l} \\ a_j - b_{l,j} + v_{l,j} & \leq M(1 - \nu_{a,j}^{l}) \end{cases}$$

Linear programming - Constraints modeling

Condition to assign an alternative a in category C_h

$$a \in C_h \iff \begin{cases} \sum_{j=1}^n c_{a,j}^{h-1} \geq \lambda & \text{and } \sum_{j=1}^n \mu_{a,j}^{h-1} < \Lambda \\ \sum_{j=1}^n c_{a,j}^{h} < \lambda & \text{or } \sum_{j=1}^n \mu_{a,j}^{h} \geq \Lambda \end{cases}$$

with $c_{a,j}^l$ and $\mu_{a,j}^l$ for $l = h-1, h$ such that :

$$c_{a,j}^l = \begin{cases} w_j & \text{if } a_j \geq b_{l,j} \\ 0 & \text{if } a_j < b_{l,j} \end{cases} \qquad \mu_{a,j}^l = \begin{cases} z_j & \text{if } a_j \leq b_{l,j} - v_{l,j} \\ 0 & \text{if } a_j > b_{l,j} - v_{l,j} \end{cases}$$

To linearize these constraints, we introduce binary variables :

$$\delta_{a,j}^l = \begin{cases} 1 & \text{if } a_j \geq b_{l,j} \\ 0 & \text{if } a_j < b_{l,j} \end{cases} \qquad \nu_{a,j}^l = \begin{cases} 1 & \text{if } a_j \leq b_{l,j} - v_{l,j} \\ 0 & \text{if } a_j > b_{l,j} - v_{l,j} \end{cases}$$

$$\begin{cases} c_{a,j}^l & \leq \delta_{a,j}^l \\ c_{a,j}^l & \leq w_j \\ c_{a,j}^l & \geq \delta_{a,j}^l - 1 + w_j \end{cases} \qquad \begin{cases} \mu_{a,j}^l & \leq \nu_{a,j}^l \\ \mu_{a,j}^l & \leq z_j \\ \mu_{a,j}^l & \geq \delta_{a,j}^l - 1 + z_j \end{cases}$$

Linear programming - Constraint modeling

Objective function

1. Maximize number of alternatives compatible with the model
2. Minimize the number of vetoes

We introduce new binary variables :

$$\gamma_a = \begin{cases} 1 & \text{if } a \text{ is assigned in the right category} \\ 0 & \text{if } a \text{ is assigned in a wrong category} \end{cases}$$

$$\omega_a^l = \begin{cases} 1 & \text{if veto applies for alternative } a \text{ against profile } l \\ 0 & \text{otherwise} \end{cases}$$

Linear programming - Constraint modeling

Objective function

1. Maximize number of alternatives compatible with the model
2. Minimize the number of vetoes

We introduce new binary variables :

$$\gamma_a = \begin{cases} 1 & \text{if } a \text{ is assigned in the right category} \\ 0 & \text{if } a \text{ is assigned in a wrong category} \end{cases}$$

$$\omega_a^l = \begin{cases} 1 & \text{if } \sum_{j=1}^n \mu_{a,j}^l \geq \Lambda \\ 0 & \text{if } \sum_{j=1}^n \mu_{a,j}^l < \Lambda \end{cases} \Rightarrow \begin{cases} \sum_{j=1}^n \mu_{a,j} - \Lambda \geq M(\omega_a^h - 1) \\ \sum_{j=1}^n \mu_{a,j} - \Lambda < M\omega_a^h \end{cases}$$

Linear programming - Constraint modeling

Objective function

1. Maximize number of alternatives compatible with the model
2. Minimize the number of vetoes

We introduce new binary variables :

$$\gamma_a = \begin{cases} 1 & \text{if } a \text{ is assigned in the right category} \\ 0 & \text{if } a \text{ is assigned in a wrong category} \end{cases}$$

$$\omega_a^l = \begin{cases} 1 & \text{if } \sum_{j=1}^n \mu_{a,j}^l \geq \Lambda \\ 0 & \text{if } \sum_{j=1}^n \mu_{a,j}^l < \Lambda \end{cases} \Rightarrow \begin{cases} \sum_{j=1}^n \mu_{a,j} - \Lambda \geq M(\omega_a^h - 1) \\ \sum_{j=1}^n \mu_{a,j} - \Lambda < M\omega_a^h \end{cases}$$

Finally :

$$a \in C_h \iff \begin{cases} \sum_{j=1}^n c_{a,j}^{h-1} - \omega_a^{h-1} \geq \lambda + M(\gamma_a - 1) \\ \sum_{j=1}^n c_{a,j}^h - \omega_a^h < \lambda - M(\gamma_a - 1) \end{cases}$$

$$\max \sum_{a \in A} \gamma_a - \frac{1}{2\,|a \in A \backslash A_1|} \sum_{a \in A \backslash A_1} \omega_a^{h-1} - \frac{1}{2\,|a \in A \backslash A_p|} \sum_{a \in A \backslash A_p} \omega_a^h \qquad (1)$$

Linear programming - MIP

$$\max \sum_{a \in A} \gamma_a - \frac{1}{2\,|a \in A \backslash A_1|} \sum_{a \in A \backslash A_1} \omega_a^{h-1} - \frac{1}{2\,|a \in A \backslash A_p|} \sum_{a \in A \backslash A_p} \omega_a^{h}$$

$$\begin{cases}
\sum_{j=1}^n c_{a,j}^{h-1} - \omega_a^{h-1} & \geq & \lambda + M(\gamma_a - 1) & \forall a \in A_h, \forall h \in H \\
\sum_{j=1}^n c_{a,j}^{h} - \omega_a^{h} & < & \lambda - M(\gamma_a - 1) & \forall a \in A_h, \forall h \in H \\
a_j - b_{l,j} & < & M\delta_{a,j}^l & \forall a \in A_h, \forall h \in H, l = \{h-1, h\} \setminus \{0, p\}, \forall j \in F \\
a_j - b_{l,j} & \geq & M(\delta_{a,j}^l - 1) & \forall a \in A_h, \forall h \in H, l = \{h-1, h\} \setminus \{0, p\}, \forall j \in F \\
a_j - b_{l,j} + v_{l,j} & > & -M\nu_{a,j}^l & \forall a \in A_h, \forall h \in H, l = \{h-1, h\} \setminus \{0, p\}, \forall j \in F \\
a_j - b_{l,j} + v_{l,j} & \leq & M(1 - \nu_{a,j}^l) & \forall a \in A_h, \forall h \in H, l = \{h-1, h\} \setminus \{0, p\}, \forall j \in F \\
c_{a,j}^l & \leq & \delta_{a,j}^l & \forall a \in A_h, \forall h \in H, l = \{h-1, h\} \setminus \{0, p\}, \forall j \in F \\
c_{a,j}^l & \leq & w_j & \forall a \in A_h, \forall h \in H, l = \{h-1, h\} \setminus \{0, p\}, \forall j \in F \\
c_{a,j}^l & \geq & \delta_{a,j}^l - 1 + w_j & \forall a \in A_h, \forall h \in H, l = \{h-1, h\} \setminus \{0, p\}, \forall j \in F \\
\mu_{a,j}^l & \leq & \nu_{a,j}^l & \forall a \in A_h, \forall h \in H, l = \{h-1, h\} \setminus \{0, p\}, \forall j \in F \\
\mu_{a,j}^l & \leq & z_j & \forall a \in A_h, \forall h \in H, l = \{h-1, h\} \setminus \{0, p\}, \forall j \in F \\
\mu_{a,j}^l & \geq & \nu_{a,j}^l - 1 + z_j & \forall a \in A_h, \forall h \in H, l = \{h-1, h\} \setminus \{0, p\}, \forall j \in F \\
\sum_{j=1}^n \mu_{a,j}^l - \Lambda & \geq & M(\omega_a^l - 1) & \forall a \in A_h, \forall h \in H, l = \{h-1, h\} \setminus \{0, p\} \\
\sum_{j=1}^n \mu_{a,j}^l - \Lambda & < & M\omega_a^l & \forall a \in A_h, \forall h \in H, l = \{h-1, h\} \setminus \{0, p\} \\
\sum_{j=1}^n w_j & = & 1 & \\
\sum_{j=1}^n z_j & = & 1 & \\
b_{h,j} & \geq & b_{h-1,j} & h = \{2, \ldots, p-1\}, \forall j \in J \\
b_{h,j} - v_{h,j} & \geq & b_{h-1,j} - v_{h-1,j} & h = \{2, \ldots, p-1\}, \forall j \in J
\end{cases}$$

Experimentation - Settings

Learning set

- Students are evaluated on 5 criteria and assigned either in category *accepted* or *refused*.
- Marks and assignments of students are constructed such that a standard MR-Sort model (i.e. without veto) cannot restore the assignments.
- To be accepted, a student should :
  - have at least 10/20 in 3 of the 5 courses ;
  - all marks at least equal to 8 with possibly one exception below 8.

Experimentation - Settings

Learning set

- To be accepted, a student should :
  - have at least 10/20 in 3 of the 5 courses ;
  - all marks at least equal to 8 with possibly one exception below 8.

	1	2	3	4	5	
St. 1	9	9	9	9	11	refused
St. 2	9	9	9	11	9	refused
St. 3	9	9	9	11	11	refused
St. 4	9	9	11	9	9	refused
St. 5	9	9	11	9	11	refused
St. 6	9	9	11	11	9	refused
St. 7	9	9	11	11	11	accepted
St. 8	9	11	9	9	9	refused
St. 9	9	11	9	9	11	refused
St. 10	9	11	9	11	9	refused
St. 11	9	11	9	11	11	accepted
St. 12	9	11	11	9	9	refused
St. 13	9	11	11	9	11	accepted
St. 14	9	11	11	11	9	accepted
St. 15	9	11	11	11	11	accepted
St. 16	11	9	9	9	9	refused
St. 17	11	9	9	9	11	refused
St. 18	11	9	9	11	9	refused
St. 19	11	9	9	11	11	accepted
St. 20	11	9	11	9	9	refused
St. 21	11	9	11	9	11	accepted
St. 22	11	9	11	11	9	accepted

	1	2	3	4	5	
St. 23	11	9	11	11	11	accepted
St. 24	11	11	9	9	9	refused
St. 25	11	11	9	9	11	accepted
St. 26	11	11	9	11	9	accepted
St. 27	11	11	9	11	11	accepted
St. 28	11	11	11	9	9	accepted
St. 29	11	11	11	9	11	accepted
St. 30	11	11	11	11	9	accepted
St. 31	11	11	11	11	7	accepted
St. 32	11	11	11	7	11	accepted
St. 33	11	11	7	11	11	accepted
St. 34	11	7	11	11	11	accepted
St. 35	7	11	11	11	11	accepted
St. 36	11	11	11	7	7	refused
St. 37	11	11	7	11	7	refused
St. 38	11	7	11	11	7	refused
St. 39	7	11	11	11	7	refused
St. 40	11	11	7	7	11	refused
St. 41	11	7	11	7	11	refused
St. 42	7	11	11	7	11	refused
St. 43	11	7	7	11	11	refused
St. 44	7	11	7	11	11	refused

Experimentation - Results

- MIP is able to restore all assignments without errors
- Parameters of the model found :

	1	2	3	4	5
w_j	0.2	0.2	0.2	0.2	0.2
z_j	0.2	0.2	0.2	0.2	0.2
λ			0.6		
Λ			0.4		
$b_{1,j}$	9.0001	9.0001	9.0001	11.0000	9.0001
$vb_{1,j}$	8.9999	8.9999	7.0000	8.9999	7.0000

- Concordance profiles are located in the interval $[9.0001, 11]$
- Veto profiles are located in the interval $[7, 8.9999]$
- MR-Sort without veto restores 86% of the examples

Conclusion

- We have shown that it can be at advantage to use coalitional vetoes to model preferences
- Further steps :
  - Test more extensively MR-Sort with standard veto versus with coalitional veto
  - Design an algorithm to learn MR-Sort model with veto from large sets of examples
  - Axiomatic

Gracias por su atención !

References I

Bouyssou, D. and Pirlot, M. (2009).
An axiomatic analysis of concordance-discordance relations.
European Journal of Operational Research, 199 :468–477.

Dias, L. and Mousseau, V. (2006).
Inferring Electre's veto-related parameters from outranking examples.
European Journal of Operational Research, 170(1) :172–191.

Dias, L., Mousseau, V., Figueira, J., and Clímaco, J. (2002).
An aggregation/disaggregation approach to obtain robust conclusions with ELECTRE TRI.
European Journal of Operational Research, 138(1) :332–348.

Fortemps, P. and Słowiński, R. (2002).
A graded quadrivalent logic for ordinal preference modelling : Loyola–like approach.
Fuzzy Optimization and Decision Making, 1(1) :93–111.

References II

Leroy, A., Mousseau, V., and Pirlot, M. (2011).
Learning the parameters of a multiple criteria sorting method.
In Brafman, R., Roberts, F., and Tsoukiàs, A., editors, *Algorithmic Decision Theory*, volume 6992 of *Lecture Notes in Computer Science*, pages 219–233. Springer Berlin / Heidelberg.

Mousseau, V., Figueira, J., and Naux, J.-P. (2001).
Using assignment examples to infer weights for ELECTRE TRI method : Some experimental results.
European Journal of Operational Research, 130(1) :263–275.

Mousseau, V. and Słowiński, R. (1998).
Inferring an ELECTRE TRI model from assignment examples.
Journal of Global Optimization, 12(1) :157–174.

References III

Ngo The, A. and Mousseau, V. (2002).
Using assignment examples to infer category limits for the ELECTRE TRI method.
Journal of Multi-criteria Decision Analysis, 11(1) :29–43.

Öztürk, M. and Tsoukiàs, A. (2007).
Modelling uncertain positive and negative reasons in decision aiding.
Decision Support Systems, 43(4) :1512–1526.

Perny, P. (1998).
Multicriteria filtering methods based on concordance/non-discordance principles.
Annals of Operations Research, 80 :137–167.

Perny, P. and Roy, B. (1992).
The use of fuzzy outranking relations in preference modelling.
Fuzzy Sets and Systems, 49(1) :33–53.

References IV

Roy, B. and Słowiński, R. (2008).
Handling effects of reinforced preference and counter-veto in credibility of outranking.
European Journal of Operational Research, 188(1) :185–190.

Sobrie, O., Mousseau, V., and Pirlot, M. (2013).
Learning a majority rule model from large sets of assignment examples.
pages 336–350. Springer.

Yu, W. (1992).
Aide multicritère à la décision dans le cadre de la problématique du tri : méthodes et applications.
PhD thesis, LAMSADE, Université Paris Dauphine, Paris.